Toward a Compositional Theory of Sensor-Based Robotic Systems

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Introduction and Motivation

Sensor-based robotic systems hold the promise to make broad impact in diverse segments of society, such as health care, automotive and aeronautical industry, agriculture, security. They perform tasks such as tracking, counting, monitoring, pursuit-evasion, navigation, and coverage. One troubling aspect of these systems is the lack of a fundamental understanding and theoretical foundation that is analogous to theory of computation for computer science, which provides notions such as system power, comparison, complexity and problem solvability, and equivalence. Such a foundation would be useful for understanding and exploiting the information requirements and complexity inherent in tasks, and the relative tradeoffs in the design of systems.

Given some task, we want to know if a system is powerful enough to complete it; also, we want to compare the system against others that can complete the task. This is an important element in the design of a robotic system because it allows us find potential tradeoffs between computational, sensing, and actuation power requirements, and can bring benefits in terms of energy, price, and communication. In contrast to the theory of computation, a theory of sensor-based systems presents substantial challenges due to the interaction with the physical world and the existence of complex dependencies between its components (sensing, actuation, and computation).

Our research direction and efforts are inspired by several others. Blum and Kozen [1] showed how the task of maze searching can be performing using only logarithmic space, as opposed requiring full SLAM. Mason and Erdmann have emphasized the importance of finding the minimal information requirements necessary to achieve tasks, in the context of manipulation [5]. In [3] a methodology was discussed to create complex robotics systems from simple units in actuation and sensing. Donald [4] provided a framework based on information invariants that enables the comparison of sensor systems by addition, deletion, and reallocation of computation, sensing, actuation, and communication. In [7] a dominance relationship based on the description of a robot as a set of primitives was proposed. A similar hierarchy of robotic systems that perform tasks in a polygonal environment was presented in [2].

In the ongoing efforts of our research group, we have been developing sensor-centric tools and concepts that may ultimately help in designing better planning algorithms. See [6] for a recent perspective. Information spaces arising from sensors and filters appear to be the natural counterpart to the crucial C-space in motion planning. By carefully studying sensor mappings and their induced partitions of state spaces, we have shown how to construct reduced-complexity filters over small information spaces. We have introduced the notion of sensor lattices, as a way to compare sensor power and understand the relative complexity tradeoffs when sensors are interchanged. This has led us to wonder whether a compositional theory can be developed in which complicated systems are formed from simple sensing, actuation, and computation primitives. We describe some of these tentative ideas by a simple example in the next part.

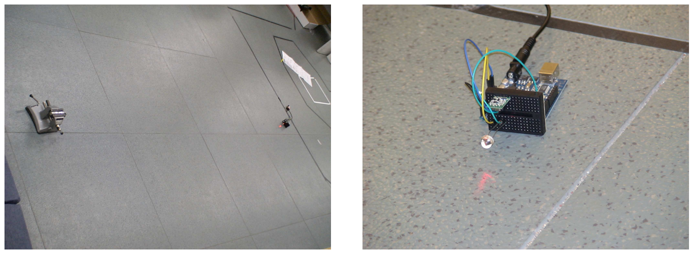

Fig. 1. A simple hardware implementation of a occupancy beam using an inexpensive laser, a photo diode and a 8-bit microcontroller

An example

Suppose that we have several agents (robots or humans) moving in an environment among obstacles. We start with a primitive sensor system: A simple occupancy sensor beam, like the one used in garage doors, that has a line as its detection (or visibility) region. This sensor can be implemented in several modalities, like a PIR, an IR or sonar, a camera, among others(see Figure 1). We call this an *occupancy beam* (see Figure 2(a)).

By adding some simple computation, we can convert this occupancy beam into a *crossing beam* which indicates that an agent just crossed it, by detecting a change in the output of the occupancy beam (see Figure 2(b)). In addition, we can put two crossing beams close together to get a *directional beam*, using the order of activation of the crossing beams (see Figure 2(c)).

Now suppose that we several agents move in an environment among obstacles, as illustrated in the Figure 3. We can parti-

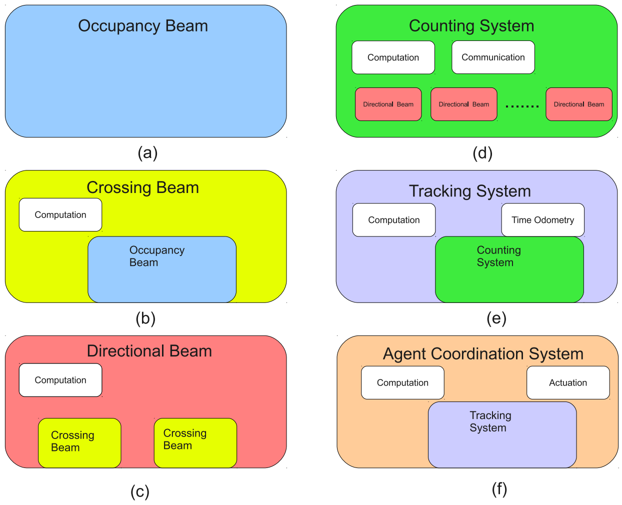

Fig. 2. Increasingly complex sensor-based systems are built from simple primitives.

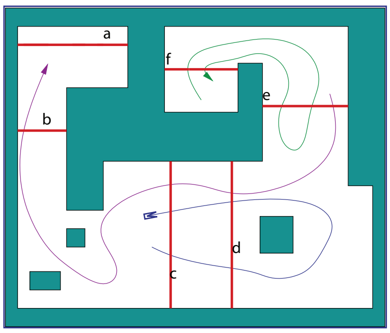

Fig. 3. Three agents moving around an environment with obstacles

tion the environment into regions of interest with directional beams labeled a, b, c, d, e, and f. For example, in Figure 3, left-to-right and bottom-to-top are the forward directions. The concatenation of the observations over some period of time can be encoded as a string: $\tilde{y} = e^{-1}d^{-1}c^{-1}bcdd^{-1}c^{-1}fe^{-1}ef^{-1}$, in which letters indicate the particular beam crossed and -1 indicates the reverse direction.

We have recently shown that given an initial condition and the observation string $\tilde{y}$, we can estimate the number of agents for each region, yielding a *counting system* (see Figure 2(d)). We are also developing algorithms to return a set of possible trajectories for each individual agent $\tilde{y}_i$, resulting in a *tracking system* (see Figure 2(e)). Now suppose we add a simple actuator for each beam that induces a direction of motion. Given an initial condition, we can give a plan $\pi : \mathcal{I} \to U$ from an appropriate information space into the set of actions that will guide the agents to a desired configuration in the workspace. This creates an *agent coordination system* (see Figure 2(f)). Some fundamental issues are explained next.

Fundamental Issues

Our ideas are rooted in the studies of sensor mappings and their preimages. Let X denote a *state space*, in which each state $x \in X$ characterizes the world at some instant of time. This state is available as an input to a sensor which returns an observation y, where $y \in Y$ for some *observation space*. The ideal sensor can be characterized by a *sensor mapping* $h : X \to Y$.

Given a sensor mapping $h : X \to Y$, for any observation $y \in Y$ its preimage can be defined as follows:

$$h^{-1}(y) = \{x \in X \mid y = h(x)\}, \tag{1}$$

This corresponds to the set of all $x \in X$ for which the sensor produces the same observation; providing a resolution at which X can be sensed, and partitioning the X into equivalence classes. Let $\Pi(h)$ be the partition induced by the sensor mapping h.

In this scenario, we have a natural way to compare sensor mappings. With X fixed, let $h_1 : X \to Y_1$ and $h_2 : X \to Y_2$ be two sensor mappings, we say that h_1 dominates h_2 if and only if $\Pi(h_1)$ is a refinement of $\Pi(h_2)$. In other words, there exits a function g that maps the observations of the sensor h_1 into observations of h_2:

$$\begin{tikzcd} X \arrow[rr, "h_2"] \arrow[dr, "h_1"'] & & Y_2 \\ & Y_1 \arrow[ur, "g"'] & \end{tikzcd}.$$

In particular, if $g : Y_1 \to Y_2$ is computable and has polynomial complexity in time and space and h_1 can be implemented with a low-cost low-energy sensor, we can replace sensor h_2 with $g \circ h_1$ as shown in Figure 2(b) where we obtained a crossing sensor h_2 from a cheap occupancy sensor h_1 and simple computation g. Moreover, we are exploring *composability*, using two crossing sensors to form a directional sensor $g_3 : Y_2 \times Y_2 \to Y_3$ and using a set of directional sensors to create a counting sensor $g_4 : Y_3 \times Y_3 \times \ldots \times Y_3 \to Y_4$ (see Figures 2(c) and (d))

By composing systems in this way, we propose to study their relative power and complexity tradeoffs. This leads to many interesting, fundamental, open questions: What components are necessary for particular tasks? How does complexity change when one system simulates another?

References

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